

1 Numbered Balls

Suppose you have a bag containing seven balls numbered 0, 1, 1, 2, 3, 5, 8.

- (a) You perform the following experiment: pull out a single ball and record its number. What is the expected value of the number that you record?

- (b) You repeat the experiment from part (a), except this time you pull out two balls together and record their total. What is the expected value of the total that you record?

2 How Many Queens?

You shuffle a standard 52-card deck, before drawing the first three cards from the top of the pile. Let X denote the number of queens you draw.

- (a) What is $\mathbb{P}(X = 0)$?
- (b) What is $\mathbb{P}(X = 1)$?
- (c) What is $\mathbb{P}(X = 2)$?
- (d) What is $\mathbb{P}(X = 3)$?
- (e) Do the answers you computed in parts (a) through (d) add up to 1, as expected?
- (f) Compute $\mathbb{E}(X)$ from the definition of expectation.

- (g) Suppose we define indicators X_i , $1 \leq i \leq 3$, where X_i is the indicator variable that equals 1 if the i th card is a queen and 0 otherwise. Compute $\mathbb{E}(X)$ using linearity of expectation.

(h) Are the X_i indicators independent? Does this affect your solution to part (g)?

3 More Aces in a Deck

There are four aces in a deck. Suppose you shuffle the deck; define the random variables:

X_1 = number of non-ace cards before the first ace

X_2 = number of non-ace cards between the first and second ace

X_3 = number of non-ace cards between the second and third ace

X_4 = number of non-ace cards between the third and fourth ace

X_5 = number of non-ace cards after the fourth ace

1. What is $X_1 + X_2 + X_3 + X_4 + X_5$?
2. Argue that the X_i random variables all have the same distribution. Are they independent?

3. Use the results of the previous parts to compute $\mathbb{E}(X_1)$.