1 Stable Marriage

Consider the set of men $M = \{1, 2, 3\}$ and the set of women $W = \{A, B, C\}$ with the following preferences.

Men	Women
1	A > B > C
2	B > A > C
3	A > B > C

Women	Men
A	2 > 1 > 3
B	1 > 2 > 3
C	1 > 2 > 3

Run the traditional propose-and-reject algorithm on this example. How many days does it take and what is the resulting pairing? (Show your work.)

2 Universal Preference

Suppose that preferences in a stable marriage instance are universal: all n men share the preferences $W_1 > W_2 > \cdots > W_n$ and all women share the preferences $M_1 > M_2 > \cdots > M_n$.

(a) What pairing do we get from running the algorithm with men proposing? Can you prove this happens for all n?

(b) What pairing do we get from running the algorithm with women proposing?

(c) What does this tell us about the number of stable pairings?

3 Propose-and-Reject Proofs

Prove the following statements about the traditional propose-and-reject algorithm.

(a) In any execution of the algorithm, if a woman receives a proposal on day i, then she receives some proposal on every day thereafter until termination.

(b) In any execution of the algorithm, if a woman receives no proposal on day i, then she receives no proposal on any previous day j, $1 \leq j < i$.

(c) In any execution of the algorithm, there is at least one woman who only receives a single proposal. (Hint: use the parts above!)