CS 70 Discrete Mathematics and Probability Theory
Fall 2018 Alistair Sinclair and Yun Song

DIS 07B

1 Monty Hall Challenge

Let us take on the challenge posed in lecture, and formally analyze the Monty Hall Problem.

(a) Assume that the corgi (the prize) and two goats were placed uniformly at random behind the three doors. What is the probability space $(\Omega, \mathbb{P})$?

(b) If our contestant chose door 1 in the first round, and decides to switch to another door after being shown a goat behind door 2 or 3, what are the events $C_1 =$"They win the corgi" and $\overline{C_1} =$"They win a goat"? What are their probabilities $\mathbb{P}(C_1)$ and $\mathbb{P}(\overline{C_1})$?

(c) If the contestant does not switch doors, what are the events $C_2, \overline{C_2}$ of winning the corgi and goats, and their respective probabilities now?

(d) If instead of choosing door 1 in the beginning, they chose a door uniformly at random, how do your $\Omega, \mathbb{P}, C_i, \overline{C_i}$ from above change?

2 Probability Warm-Up

(a) Suppose that we have a bucket of 30 red balls and 70 blue balls. If we pick 20 balls out of the bucket, what is the probability of getting exactly k red balls (assuming $0 \le k \le 20$) if the sampling is done with replacement?

(b) Same as part (a), but the sampling is without replacement.

(c) If we roll a regular, 6-sided die 5 times. What is the probability that at least one value is observed more than once?

3 Polynomial Probabilities

(a) Let us pick a degree $< p$ polynomial f over $\mathrm{GF}(p)$ uniformly at random. What is the probability space $(\Omega, \mathbb{P})$?

(b) What is the probability that $f(0) = a$ for some fixed $a \in \mathrm{GF}(p)$?

(c) Assume Alice shared a secret with $\mathrm{Bob}_1, \mathrm{Bob}_2$ and Bob_3. That is, she constructed a polynomial g of degree at most 2 with $p(0) = s$. If Bob_1 and Bob_2 got together and made a (uniform) random guess at what Bob_3's value was, what is the probability that they recover s correctly?