CS 70 Discrete Mathematics and Probability Theory Fall 2018 Alistair Sinclair and Yun Song Discussion 08B

1 Let's Talk Probability

- (a) When is $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$ true? What is the general rule that always holds?
- (b) When is $\mathbb{P}(A \cap B) = \mathbb{P}(A) * \mathbb{P}(B)$ true? What is the general rule that always holds?
- (c) If A and B are disjoint, are they independent?

2 Aces

Consider a standard 52-card deck of cards:

- (a) Find the probability of getting an ace or a red card, when drawing a single card.
- (b) Find the probability of getting an ace or a spade, but not both, when drawing a single card.
- (c) Find the probability of getting the ace of diamonds when drawing a 5 card hand.
- (d) Find the probability of getting exactly 2 aces when drawing a 5 card hand.
- (e) Find the probability of getting at least 1 ace when drawing a 5 card hand.
- (f) Find the probability of getting at least 1 ace or at least 1 heart when drawing a 5 card hand.

3 Balls and Bins

Throw n balls into n bins.

- (a) What is the probability that the first bin is empty?
- (b) What is the probability that the first *k* bins are empty?
- (c) Use the union bound to give an upper bound on the probability that at least k bins are empty.
- (d) What is the probability that the second bin is empty given that the first one is empty?
- (e) Are the events that "the first bin is empty" and "the first two bins are empty" independent?
- (f) Are the events that "the first bin is empty" and "the second bin is empty" independent?