1   Rolling Dice

(a) If we roll a fair 6-sided die, what is the expected number of times we have to roll before we roll a 6? What is the variance?

(b) Suppose we have two independent, fair n-sided dice labeled Die 1 and Die 2. If we roll the two dice until the value on Die 1 is smaller than the value on Die 2, what is the expected number of times that we roll? What is the variance?

(c) Let $n = 6$, so we are back to fair 6-sided die. Suppose we roll Die 1 until a 6 comes up, and we roll Die 2 until a 6 comes up. Let X be a random variable representing the number of times Die 1 is rolled before getting a 6, and let Y be the corresponding random variable for Die 2. Compute $\mathbb{P}[\min(X,Y) = n]$ and $\mathbb{P}[X + Y = n]$, where n is an integer.

2   Trick or Treat

Shreyas and Jerry are trick or treating together, and are each trying to collect all n flavors of Laffy Taffy. At each house, they each receive a Laffy Taffy, chosen uniformly at random from all flavors. However, Shreyas will throw a tantrum if Jerry gets a flavor he doesn't, so they agree that if they receive different flavors, they'll both politely return the candy they got, and move on to the next house. What is the expected number of houses they need to visit until they get a full set of Laffy Taffy?

3   Unreliable Servers

In a single cluster of a Google competitor, there are a huge number of servers n, each with a uniform and independent probability of going down in a given day. On average, 4 servers go down in the cluster per day. Recall that as $n \to \infty$, a $\text{Binom}(n, \lambda/n)$ distribution will tend towards a $\text{Poisson}(\lambda)$ distribution.

(a) What is an appropriate distribution by which the number of servers that crash can be modeled?

(b) Compute the expected value and variance of the number of crashed servers for a certain cluster.

(c) Compute the probability that fewer than 3 servers crashed.

(d) Compute the probability at least 3 servers crashed.