CS 70   Discrete Mathematics and Probability Theory

Fall 2018   Alistair Sinclair and Yun Song   HW 10

Due: Monday, November 5, 2018 at 10 PM

Sundry

Before you start your homework, state briefly how you worked on it. Who else did you work with?
List names and email addresses. (In case of homework party, you can just describe the group.)

1 Family Planning

Mr. and Mrs. Brown decide to continue having children until they either have their first girl or until
they have three children. Assume that each child is equally likely to be a boy or a girl, independent
of all other children, and that there are no multiple births. Let G denote the numbers of girls that
the Browns have. Let C be the total number of children they have.

(a) Determine the sample space, along with the probability of each sample point.

(b) Compute the joint distribution of G and C. Fill in the table below.

	$C = 1$	$C = 2$	$C = 3$
$G = 0$			
$G = 1$			

(c) Use the joint distribution to compute the marginal distributions of G and C and confirm that
the values are as you'd expect. Fill in the tables below.

$\mathbb{P}(G = 0)$	
$\mathbb{P}(G = 1)$	

$\mathbb{P}(C = 1)$	$\mathbb{P}(C = 2)$	$\mathbb{P}(C = 3)$

(d) Are G and C independent?

(e) What is the expected number of girls the Browns will have? What is the expected number of
children that the Browns will have?

2 Will I Get My Package?

A delivery guy in some company is out delivering n packages to n customers, where $n \in \mathbb{N}$, $n > 1$. Not only does he hand a random package to each customer, he opens the package before delivering it with probability $1/2$. Let X be the number of customers who receive their own packages unopened.

(a) Compute the expectation $\mathbb{E}(X)$.

(b) Compute the variance $\text{var}(X)$.

3 Double-Check Your Intuition Again

(a) You roll a fair six-sided die and record the result X. You roll the die again and record the result Y.

   (i) What is $\text{cov}(X+Y, X-Y)$?

   (ii) Prove that $X+Y$ and $X-Y$ are not independent.

For each of the problems below, if you think the answer is "yes" then provide a proof. If you think the answer is "no", then provide a counterexample.

(b) If X is a random variable and $\text{var}(X) = 0$, then must X be a constant?

(c) If X is a random variable and c is a constant, then is $\text{var}(cX) = c\,\text{var}(X)$?

(d) If A and B are random variables with nonzero standard deviations and $\text{Corr}(A,B) = 0$, then are A and B independent?

(e) If X and Y are not necessarily independent random variables, but $\text{Corr}(X,Y) = 0$, and X and Y have nonzero standard deviations, then is $\text{var}(X+Y) = \text{var}(X) + \text{var}(Y)$?

(f) If X and Y are random variables then is $\mathbb{E}(\max(X,Y)\min(X,Y)) = \mathbb{E}(XY)$?

(g) If X and Y are independent random variables with nonzero standard deviations, then is

$$\text{Corr}(\max(X,Y), \min(X,Y)) = \text{Corr}(X,Y)?$$